SUBJECT : MATHEMATICS (SET-I)

Time : 3 Hrs. **M.M.: 100**

General Instructions :

(i) All questions are compulsory.

(ii) Question Nos. 1 to 6 are of 1 mark each.

(iii) Question Nos. 7 to 19 are of 4 marks each.

(iv) Question Nos. 20 to 26 are of 6 marks each.

(v) Use of calculators is not permitted.

SECTION-A

Q1. Find the domain of the function $f(x) = -|x-2|$

Q2. Find the value of $\cos(-1710°)$

Q3. Find the value of n, if ${}^{n}P_4 : {}^{n}P_5 = 1:2$

Q4. How many chords can be drawn through 21 points on a circle?

Q5. Find the term independent of x in the expansion of

$$\left(2x - \frac{1}{x}\right)^{10}, \; x \neq 0$$

Q6. Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 2x}$

SECTION-B

Q7. (a) Find the derivative of $f(x) = (5x^3 + 3x - 1)(x - 1)$

(b) Using Binomial Theorem, find the value of $(99)^4$.

Q8. Find the domain and range of the function $f(x) = \sqrt{x^2 - 4}$

Q9. If $R = \{(x, y) : x, y \in W, 2x + y = 8\}$ then

(a) find the domain and the range of R.

(b) Write R as a set of Ordered pairs.

Q10. Using properties of sets prove the following :

(i) $(A \cap B) \cup (A - B) = A$

(ii) $A \cup (B - A) = A \cup B$

Q11. A wheel of rail engine is rotating at 1200 rotations per minute. If the radius of wheel is 35 cm. What linear distance does a point of its rim travelled in 30 seconds. What role does railways play in India's transportation system especially for goods?

Q12. Prove that:
$\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$

Q13. Using PMI, prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133 for all natural numbers n.

Q14. Prove that $\sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x$

Q15. Solve the following system of linear inequalities and represent the solution on the number line.

$$2(2x+3) - 10 < 6(x-2); \quad \frac{2x-3}{4} + 6 \geq 4 + \frac{4x}{3}$$

Q16. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) atleast one boy and one girl (ii) atleast two girls. Explain the importance of teamwork in achieving success in life.

Q17. Find the coefficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$

Q18. Find the derivative of $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$

Q19. Find the derivative of $f(x) = \sin(2x + 3)$ from first principle.

SECTION-C

Q20. A T.V. Survey gives the following data for TV watching : 60% watch program A, 50% watch program B; 47% watch program C; 28% watch programs A and B; 23% watch programs A and C; 18% watch programs B and C, 8% watch programs A, B and C. Draw a venn diagram to illustrate this information and find

(a) What percentage watch programs A and B but not C?

(b) What percentage watch exactly two programs?

(c) What percentage do not watch any program? Do you think that to some extent parents should monitor TV viewing habits of children. If yes, then why?

Q21. Solve : $\cos 3x + \cos x - \cos 2x = 0$

Q22. By Mathematical Induction, prove that for all $n \in N$

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Q23. Solve the following system of inequalities graphically :

$2x + y \leq 24$

$x + y < 11$

$2x + 5y \leq 40$

$x > 0, y \geq 0$

Q24. Find the number of arrangements which can be made from the letters of the word INDEPENDENCE. In how many of these arrangements (i) words start with P; (ii) words start with I and end with P; (iii) all the vowels occur together. List the values to be promoted to maintain our independence.

Q25. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:7:42, find n.

Q26. Evaluate (any two):

(i) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

(ii) $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

(iii) $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$